USE OF ANALYTIC ELEMENT METHOD IN SAGA GROUNDWATER AQUIFER

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1. INTRODUCTION

Groundwater plays an important role in water resources management. In many regions it is the only source of water for various uses. The large volume of groundwater aquifer can serve as a storage reservoir which can supply water during drought periods. Using the techniques of artificial recharges, groundwater may serve as a natural filter plant and distribution system. Seasonal fluctuations in groundwater levels and storage volume are fairly small compared with surface water resources. While regulations of surface water resources require costly hydraulic structures such as dams, weirs and storage ponds, regulations of groundwater resource can be made by some appropriate management schemes such as proper distribution of groundwater extraction wells and recharge wells. Usually, surface water resource development requires huge investment at the biginning phase, while groundwater resource development can be made stage by stage as needs arise. As regards water quality, groundwater quality is normally better than surface water quality because it is less susceptible to man-made pollution and due to the natural purification process-subsurface filtration. Many groundwater wells provide water of good quality which can be directly consumed without any treatment.

Groundwater resource management includes determination of appropriate locations, rates and time of pumping or artificial recharging. Data on aquifer properties, flow patterns, water levels and water quality are necessary for the management. Besides field observation and investigation, a number of mathematical models have been used for studying groundwater flow patterns and pollutant dispersion. Most groundwater models are fomulated based on the Darcy's law which states that the average flow velocity is directly proportional to the hydraulic gradient. The Dupuit's assumption has often been employed to obtain simplified formulations. Analytic solutions for unconfined and confined flows into extraction wells or from recharging wells can be derived by making use of the superposition theorem and the method of images. Furthermore, many numerical models have been developed based on the Darcy's law as well as the conservation of mass equation. The numerical methods normally used in model development include the finite defference method and the finite element method. More recently, the so-called analytic element method has been developed by Strack, O.D.L.at the University of Minnesota. This method is based on superposition of suitable closed-form analytic functions which are constructed to model a particular feature in an efficient way. The freedom in the combination of elements enables refinements in an exiting model and coupling of models. It is usually applied to an infinite aquifer with variations in values of hydrologic parameters. This method was used to model groundwater flows in some regions in the U.S.A. (Strack et al, 1980; Strack, 1982; etc.) and was chosen for the construction of the national groundwater model of the Netherlands (De Lange, 1991). A comprehensive review on the analytic element method can be found in Strack (1989).

In this study, an attempt has been made to apply the analytic element method to model groundwater flow in Saga groundwater aquifer. The main purpose is to evaluate the regional flow pattern which is important information for groundwater management as well as assessment of environmental impacts related to groundwater extraction in Saga area.

2. BASIC EQUATIONS

Groundwater flow in shallow aquifers is mainly horizontal. Therefore, it can be can be simulated in the two-dimensional plane in which the mass balance equation is written as:

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial_y}=-\tau \tag{1}$$

where Q_x, Q_y are specific discharges in the x-and y-directions; τ is vertical outflow per unit area. For a homogeneous and isotropic aquifer, the Darcy's law is written as:

$$Q_x=-Kh\frac{\partial \phi}{\partial x}\ ;\ Q_y=-Kh\frac{\partial \phi}{\partial_y} \tag{2}$$

where K is hydraulic conductivity; h is depth contributing to groundwater flow; ϕ is piezometric head. For an unconfined aquifer, the depth h is variable and equal to the piezometric head ϕ, if ϕ is measured from the bottom of the aquifer. For a confined aquifer, the depth h is equal to the total depth H of the aquifer (Figure 1).

In both confined and unconfined aquifers, the specific discharges can be written in terms of discharge potential (or just potential) Φ as follow:

$$Q_x=-\frac{\partial \Phi}{\partial x}\ ;\ Q_y=-\frac{\partial \Phi}{\partial_y} \tag{3}$$

where $\Phi=KH\phi+C_c$ for the confined flow (4)

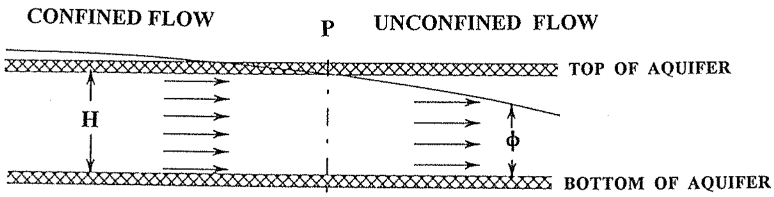


Figure 1. Scheme of Flow in Confined and Unconfined Aquifers.

and $\Phi=\frac{1}{2}K\phi^2+C_u$ for the unconfined flow (5)

Substituting Eq.(3) into Eq.(1), we obtain:

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial^2{}_y}=\tau \tag{6}$$

As mentioned above, analytic elements are described both in terms of potential function Φ and stream function Ψ. Similar to the differential equation in Φ, the differential equation in terms of stream function Ψ can be written as (Verruijt, 1982):

$$\frac{\partial^2\Psi}{\partial^2 x}+\frac{\partial^2\Psi}{\partial^2{}_y}=0 \tag{7}$$

The potential function Φ and the stream function Ψ fulfill the Cauchy-Riemann equations, i.e.

$$\frac{\partial\Psi}{\partial x}=-\frac{\partial\Phi}{\partial_y}\ ;\ \frac{\partial\Psi}{\partial_y}=\frac{\partial\Phi}{\partial x} \tag{8}$$

Therefore, Φ and Ψ may be represented as the real and imaginary parts of an analytic function $\Omega=\Omega(z)$ of the complex variable $z=x+i_y$ defined in the flow domain. This function is known as the *complex potential* Ω, i.e.

$$\Omega=\Phi+i\Psi \tag{9}$$

By using the complex potential Ω and complex variable z, mathematical expressions for the analytic elements become simpler. The complex potential functions for some types of elements are shown in the next section. Each type of element has a specific feature in groundwater flow, such as point-sinks (wells), line-sinks (rivers, canals), line-dipoles (cracks), area-sinks (infiltration areas and leaky aquifers), linedoublets (leaky walls, inhomogeneity), etc. Superposition of the complex potentials Ω of all elements leads to the complete solution. Then, at each point in the aquifer, the values of potential Φ and stream function Ψ can be computed.

3. ANALYTIC ELEMENTS

The analytic element model for groundwater flow study is developed by a combination of elements of various types. In this section, some main types of elements are introduced. More details about the analyses of complex potentials of these elements can be found in Strack (1989).

Point-Sink For a fully penetrating point-sink (a well), the complex potential can be written as:

$$\Omega_w(z)=\frac{Q_w}{2\pi}\ln\frac{(z-z_w)}{R} \tag{10}$$

where Q_w is rate of discharge of the point-sink; z_w is the location; R is the distance beyond which the effect of the point-sink is negligible.

Line-Sink A line-sink is used in the case that inflow takes place along a line and not concentrated at a point. It is used to model inflow and outflow (line-source or negative line-sink) of a river, canal, etc. of which the width is relatively small compared with its length. The line-sink can be considered as an infinite number of point-sinks along a straight line, each has a length of $d\xi$ and with an extraction rate of $\sigma d\xi$, where σ is strength per unit length (Figure 2). The complex potential contributed by a line-sink can be obtained by integrating the potential contributed by each infinitesimal strength along the line, i.e.

$$\Omega_{ls}(z)=\int_L \frac{\sigma}{2\pi}\ln(z-\delta)d\xi \tag{11}$$

where δ is a complex number denoting a point on the line.

Dipole A dipole is an element consisting of two poles, a sink and a source, with equal but opposite strengths which approach infinity at a distance that approaches zero as shown in Figure 3. The dipole is not a useful element in practice but it is the

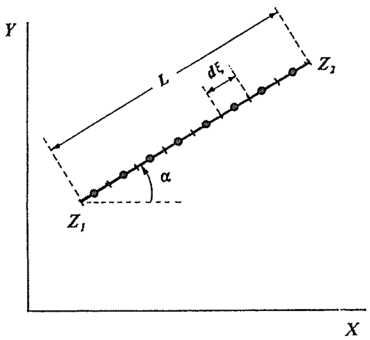


Figure 2. The Line-Sink

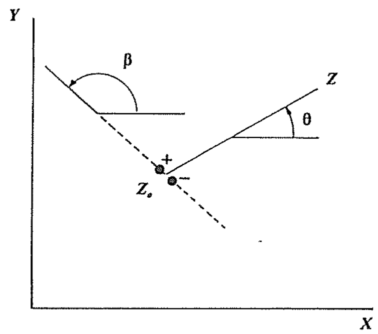


Figure 3. The Dipole

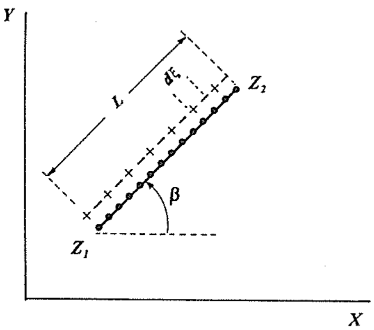


Figure 4. The Line-Dipole

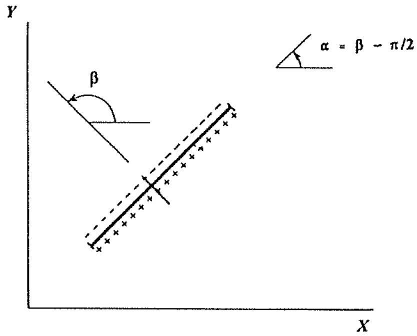


Figure 5. The Line-Doublet

basis for forming other useful elements such as line-dipole and line-doublet. The complex potential for dipole is

$$\Omega_{dp}(z)=\frac{s}{2\pi}\frac{e^{i\beta}}{z-z_o} \tag{12}$$

where s is strength of the dipole; z_o is the location; and β is orientation of the dipole.

Line-Dipole A line-dipole is a continuous distribution of dipoles along a line, with the orientation of dipoles parallel to the line (Figure 4). The complex potential of a line-dipole is obtained by integrating the potential of each dipole along the line, i.e.

$$\Omega_{lp}(z)=\frac{1}{2\pi}\int_L \frac{\mu e^{i\beta}}{z-\delta}d\xi=\frac{1}{2\pi}\int_L \frac{\mu}{z-\delta}d\delta \tag{13}$$

where μ is strength per unit length. The line-dipole is used to model a thin zone of very high hydraulic conductivity, such as a crack, in the aquifer.

Line-Doublet A line-doublet consists of an infinite number of dipoles along a line with orientation perpendicular to the line (Figure 5). This type of element is used to create a jump in the potential Φ, while the stream function Ψ remains continuous. The complex potential for a line-doublet can be found in the same way as the line-dipole, i.e.

$$\Omega_{db}(z)=\frac{i}{2\pi}\int_L \frac{\lambda e^{i\alpha}}{z-\delta}d\xi=-\frac{1}{2\pi i}\int_L \frac{\lambda}{z-\delta}d\delta \tag{14}$$

Area-Sink An area-sink element is used to model an infiltration area or leakage between adjacent aquifers. The complex potential of an area-sink may be obtained by integrating the potential of an infinitesimal point-sink over an area or by the boundary integral analysis in which the potentials and flow fields inside, outside and on the boundary are considered.

4. COMPUTATION ASPECTS

Single-Layer Model As previously mentioned, the analytic element model is obtained by superposition of all analytic elements of various features in the aquifer. If the strengths of all the elements are known, both the potential Φ and the stream function Ψ can be simply computed by adding the values contributed by each element. The general formulation for a model of analytic elements with given strengths is:

$$\Phi(z)=\sum_i\{Q_i^* U_i(z, z_i)\} \tag{15}$$

where Q_i^* is the given strength of element i; $U_i(z, z_i)$ is the potential due to element i (well, line-sink, etc.) of unit strength, which is the real part of the complex potential $\Omega_i(z)$ provided that the strength is equal to 1.0.

Generally, not all the strengths of the analytic elements are known. For example, in the case of rivers and lakes, their surface water levels are known but the discharge fluxes are unknown. In such a case, we have to formulate the system of equations and solve for the values of the unknown strengths. Assuming that there are m analytic elements with given strength and another n elements with unknown strength. Then, the potential at any point z is obtained from:

$$\Phi(z)=\sum_i^m Q_i^* U_i(z, z_i)+\sum_j^n Q_j U_j(z, z_j) \tag{16}$$

where Q_i^* is the known strength, while Q_j is the unknown strength. To solve for all Q_j, the number of observation points for Φ must be equal to the total number of unknown variables. Eq.(18) is used to compute the potential at each observation point z_k^0 and the value is set equal to the observed potential at that point Φ_k. This leads to a system of epuaions as follow:

$$\begin{aligned}
&\sum_j^n Q_j U_j(z_1^o, z_j)=\Phi_1-\sum_i^m Q_i^* U_i(z_1^o, z_i)\\
&\sum_j^n Q_j U_j(z_2^o, z_j)=\Phi_2-\sum_i^m Q_i^* U_i(z_2^o, z_i)\\
&\qquad\cdots\cdots\cdots\cdots\cdots\cdots\\
&\qquad\cdots\cdots\cdots\cdots\cdots\cdots\\
&\sum_j^n Q_j U_j(z_{n+1}^o, z_j)=\Phi_{n+1}-\sum_i^m Q_i^* U_i(z_{n+1}^o, z_i)
\end{aligned} \tag{17}$$

or in the matrix form as:

$$\boldsymbol{U}.\ \boldsymbol{Q}_u=\Phi^o-\Phi^* \tag{18}$$

where

$$\boldsymbol{Q}_u=\{Q_1, Q_2, \cdots, Q_n\} \tag{19}$$

$$\Phi^o=\{\Phi_1, \Phi_2, \cdots, \Phi_n\} \tag{20}$$

$$\Phi^*=\{\sum_i^m Q_i^* U_i(z_1^o, z_i), \sum_i^m Q_i^* U_i(z_2^o, z_i), \cdots, \sum_i^m Q_i^* U_i(z_n^o, z_i)\} \tag{21}$$

and

$$\boldsymbol{U}=\begin{bmatrix} U_1(z_1^o, z_1)\ U_2(z_1^o, z_2)\cdots U_n(z_1^o, z_n)\\ U_1(z_2^o, z_1)\ U_2(z_2^o, z_2)\cdots U_n(z_2^o, z_n)\\ \cdots\cdots\cdots\cdots\\ \cdots\cdots\cdots\cdots\\ U_1(z_n^o, z_1)\ U_2(z_n^o, z_2)\cdots U_n(z_n^o, z_n)\end{bmatrix} \tag{22}$$

Eq.(17) is a set of linear algebraic equations which can be solved by the well-known Gauss elimination method. The obtained solutions of Q_j(j=1, 2, ..., n) are then substituted in Eq.(16) to obtain the complete model for $\Phi(z)$.

Multi-Layer Model In a multi-layer aquifer, leakage area-sink elements are used to connect between layers. In case of two-layer aquifer, the system of equations can be written in the form:

$$\begin{bmatrix} \Phi^o-\Phi^*\\ (layer\ 1)\\ \Phi^o-\Phi^*\\ (layer\ 2)\end{bmatrix}=\begin{bmatrix} \boldsymbol{U}^{(1)} & \boldsymbol{V}_{12}\\ (layer\ 1) & \\ \boldsymbol{V}_{12} & \boldsymbol{U}^{(2)}\\ & (layer\ 2)\end{bmatrix}\cdot\begin{bmatrix} \boldsymbol{Q}_u^{(1)}\\ (layer\ 1)\\ \boldsymbol{Q}_u^{(2)}\\ (layer\ 2)\end{bmatrix} \tag{23}$$

in which $\boldsymbol{V}_{12}$ is the block containing the connection coefficients. A problem of three or more layers can be treated in a similar manner.

5. SAGA GROUNDWATER AQUIFERS

The Saga plain, approximately 60,000 ha in extent, is located along the northern coast of the Ariake Sea in Kyushu Island, Japan (Figure 6). It is an alluvial plain at the foot of the Sefuri and the Tenzan Mountains. The eastern part of the plain was created mainly through deposition of sediments that had been carried along the

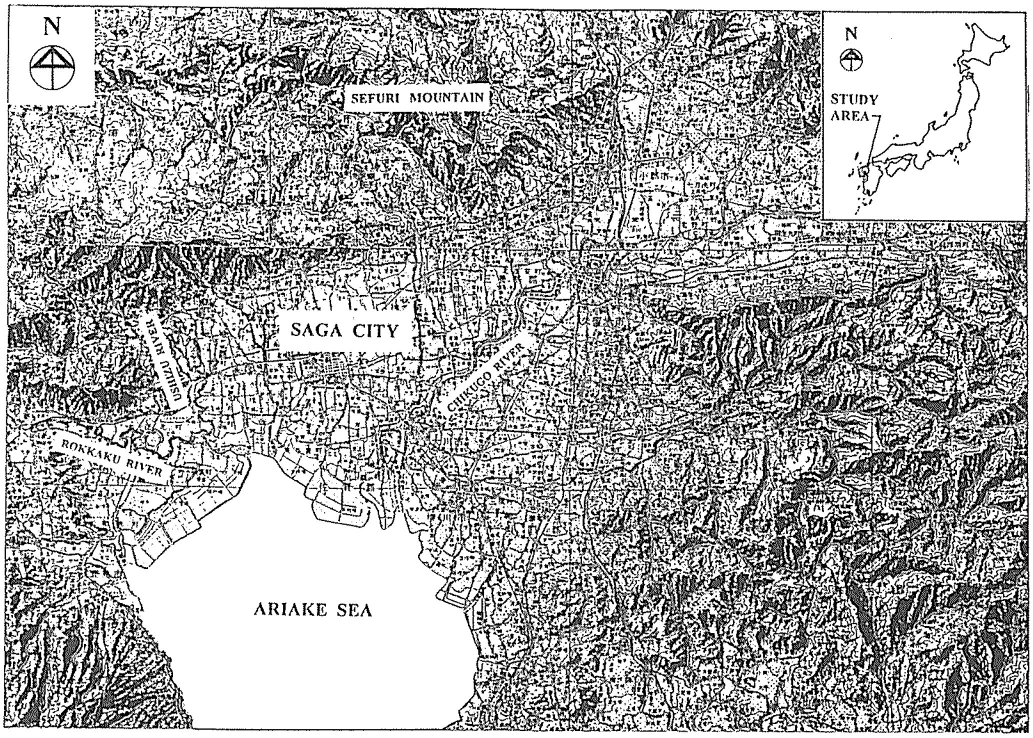


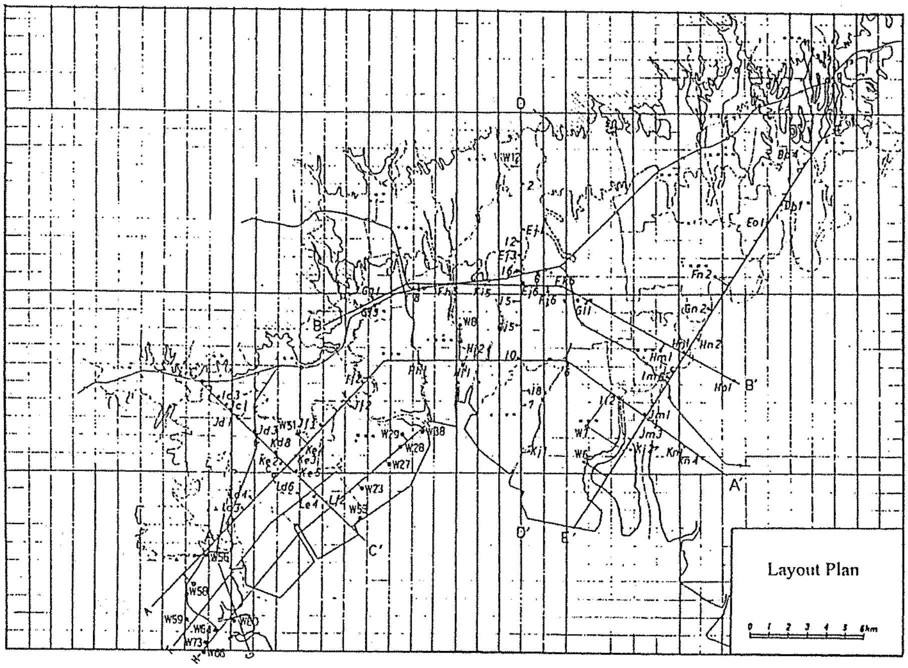
Figure 6. Location of Saga Plain

Chikugo and the Kase Rivers. The western part is also an alluvial plain formed by deposition of sediments carried down from the hilly areas along the Rokkaku and the Shiota Rivers.

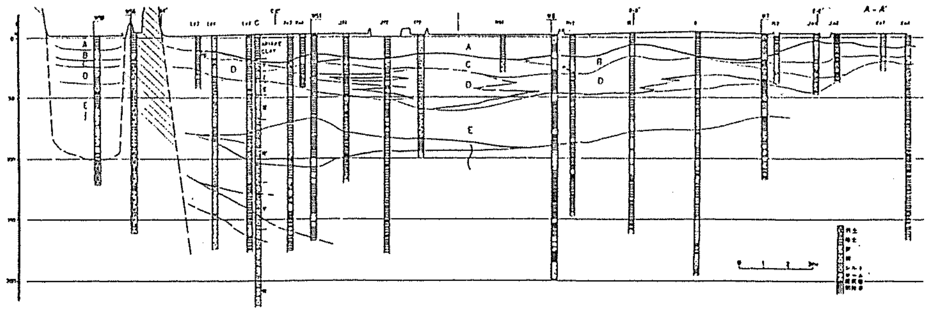
The subsurface geology of the Saga plain can be divided into 6 formations (Figure 7) denoted by A, B, C, D, E and F. The top layer (A: Ariake clay) is mainly composed of fine silt and soft clay. The thickness is usually in the range of 10-15 m with a maximum of 30 m. The second layer (B: Shimabara Bay formation) is diluvial marine deposit, mainly composed of sand. The thickness is about 10-15 m. This layer is the top aquifer where groundwater was extracted in the past. The third layer (C: the pumice-bearing volcanic ash formation) was originated from Mt. Aso as pyroclastic flow. The thickness is about 10 m. It is the well-known key bed of the top aquifer. The fourth layer (D: Nagabaru formation) is composed of clay, silt, sand and gravel. The thickness is about 30 m. The fifth layer (E: Kawazoe formation) is composed of solidified clay in the upper part with sand and gravel in the lower part. The thickness is about 100 m. It is the main aquifer of Saga plain. The bottom layer (F formation) has fairly compact silt and fossils of wood (Miura et al, 1988).

6. MODEL DEVELOPMENT

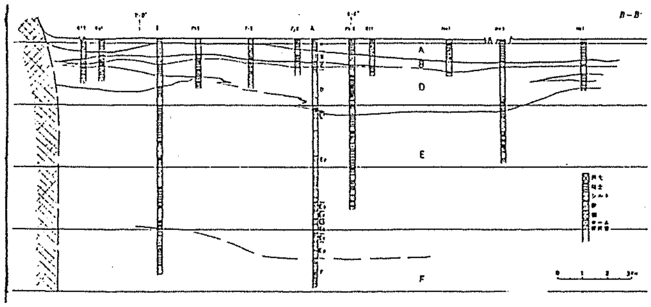
In this study the analytic element method is used to investigate the flow pattern in Saga groundwater aquifer. The modeling study is conducted step by step as



(a) **LAYOUT PLAN**

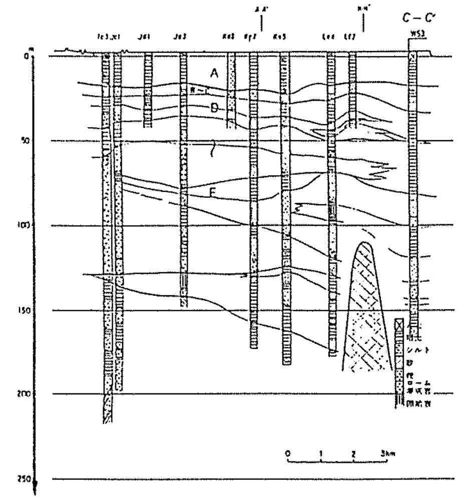


(b) **SECTION A-A′**

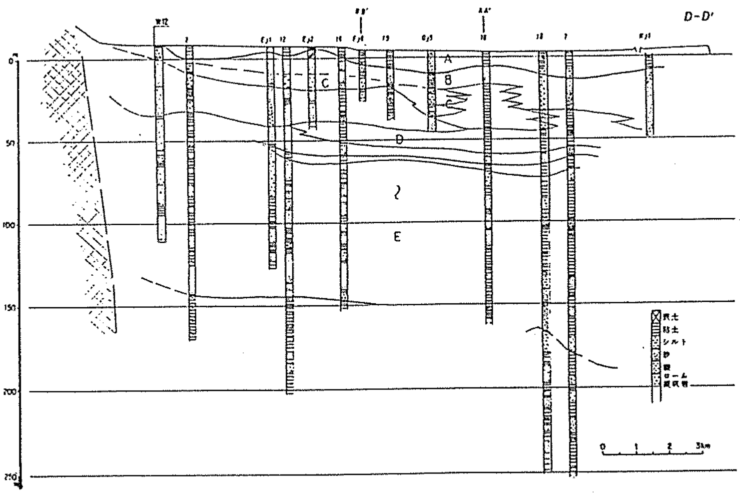


(c) **SECTION B-B′**

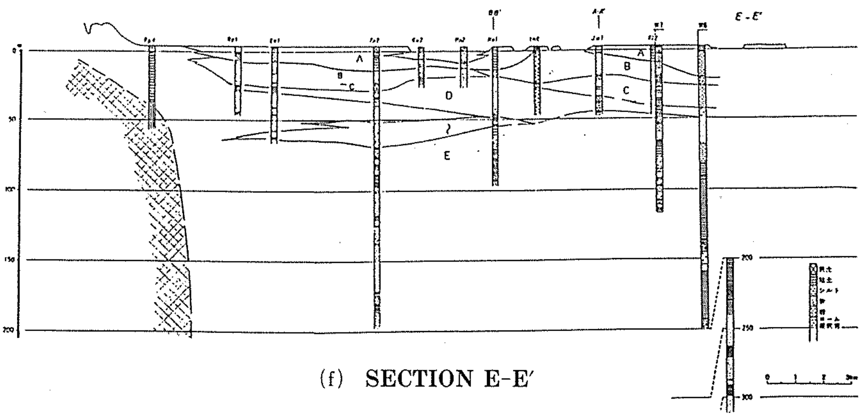
Figure 7. Geological Profile of the Saga Plain



(d) **SECTION C-C′**



(e) **SECTION D-D′**



(f) **SECTION E-E′**

Figure 7. (Cont.)

follows:

Identification of the Study Area The first step of the study is to identify the study area. This study is aimed at investigating groundwater flow pattern in Saga City and nearby areas as shown in Figure 8. In order to simulate the hydrologic effects of the adjacent areas, analytic elements in the surrounding zone are also included at a relatively coarser level. The surrounding zone considered in this study extends to the Ushizu River on the west, the Chikugo River on the east, the Sefuri Mountain on the north, and the Ariake Sea on the south.

Hydrogeological Schematization Data on topography and hydrology of the Saga plain as well as hydrogeological properties of Saga groundwater aquifers and existing groundwater wells are collected from various sources. These data are used to define the types of aquifers and to identify the types of analytic elements to be included in the model. It is found that only the lower aquifer is the main groundwater source nowadays. Therefore, it is decided to use a single-layer model in this study. The studied aquifer is a confined aquifer with semi-permeable volcanic ash layer on the upper boundary. It extends about 30-160 m from the ground surface. In fact, this lower aquifer can be further divided into 2 layers separated by thin solidified clay layer. However, this clay layer is not continuous over the whole aquifer. In order to simplify the problem, the lower aquifer is considered as one layer in this study.

Development of Coarse Model In this step, some analytic elements are introduced in order to cope with the boundary conditions of the whole study area and to simulate the effects of the main hydrological features on the boundary elements

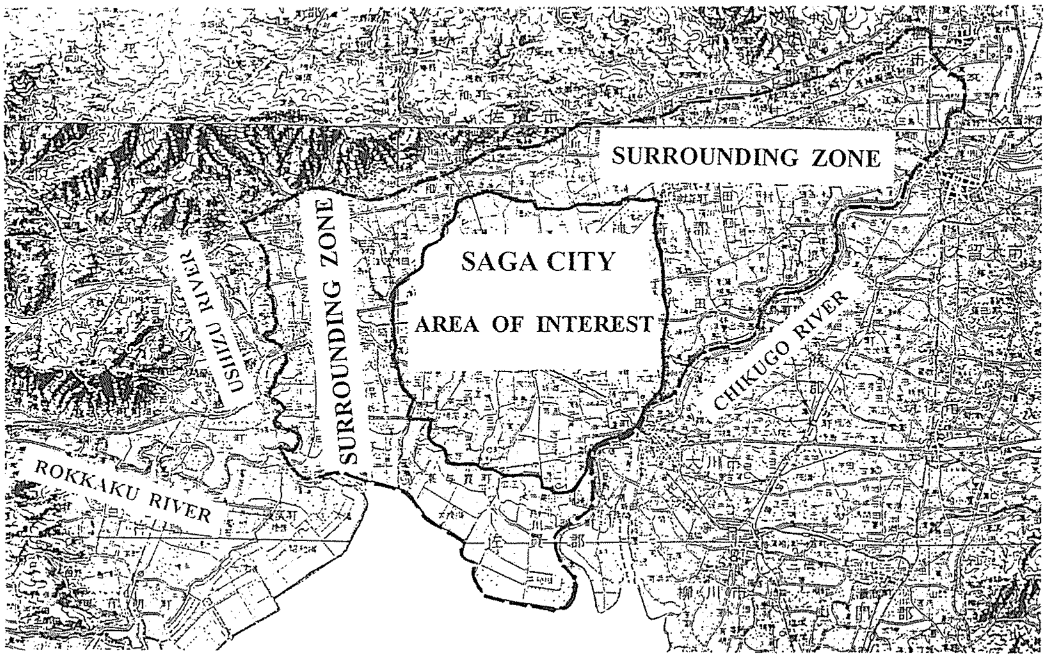


Figure 8. Area of Insterest and Surrounding Zone

with unknown strengths. In this study, the coastline on the south, the Ushizu River on the west and the Chikugo River on the southeast are simulated by series of line-sinks with specified heads. Since the northern boundary is continuously supplied with subsurface flow from the mountains, therefore, it is also simulated by another series of line-sinks. Groundwater wells located in a nearby area are combined and their pumping rates are added and considered as a single well in each subarea. Leakage from the upper aquifer distributed over the whole study area is simulated by leaky area-sinks. The rate of leakage is considered to be proportional to the difference between the discharge potentials of the upper and the lower aquifers. Since the potential in the lower aquifer is the unknown variable, an iteration technique must be employed. At the beginning, the value of potential in the lower aquifer is assumed, while the potential value in the upper aquifer is obtained from the observed data. With this assumed potential, the rate of leakage can be determined and used in computation of the unknown strengths of the boundary elements. After that, the value of potential in the lower aquifer can be computed. Then, the assumed potential is replaced by the computed value and the computation is repeated. A few iterations are required to obtain steady results. The value of vertical resistance of the semi-permeable layer separating the upper and the lower aquifers is tuned so that the computed potential in the lower aquifer is close to the observed data at some check points. The groundwater well data as well as observed piezometric heads in December 1979 are used in the calibration. Figure 9 illustrates the configuration of link-sinks and area-sinks used in the coarse model development.

Devolopment of Finer Model In order to obtain more accurate results in such

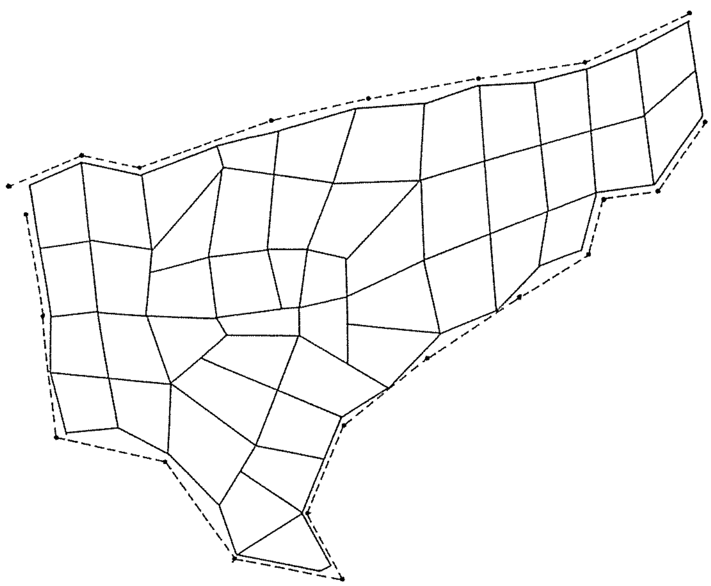


Figure 9. Configuration of Line-Sinks and Area-Sinks used in Coarse Model Development

a way that the effects of each element on groundwater flow pattern in the area of interest can be evaluated, refinement of the coarse model is undertaken emphasizing on the area of interest. Since the main hydrological elements in the Saga City area are groundwater wells, the refinement is made by subdividing the groups of groundwater wells into smaller groups. It should be noted that consideration of an individual well is not possible in this study due to the existing data collection system. Regarding the leakage from the upper aquifer, the same area-sinks as shown in Figure 9 are used in this finer model. The computation is repeated as described above using the value of vertical resistance of the semi-permeable layer that has been tuned in the previous step.

7. PRESENTATION OF RESULTS AND CONCLUSIONS

The results obtained from the model are in the form of discharge potentials Φ at various grid points in the study area. These discharge potentials are converted to piezometric heads ϕ by using Eq.(4). The piezometric contour for the lower aquifer in the whole study area computed from the finer model is plotted as shown in Figure 10.

1) Modeling with analytic elements requires a systematic approach which includes steps in modeling as well as organization of input and output.

2) The analytic element method allows for free combination of elements of different features, i.e., very large elements such as area-sinks and line-sinks can be connected to very small elements such as wells. Also, extreme variations in values of hydrologic parameters are possible.

3) Modeling with the analytic element method can be made at different accuracy

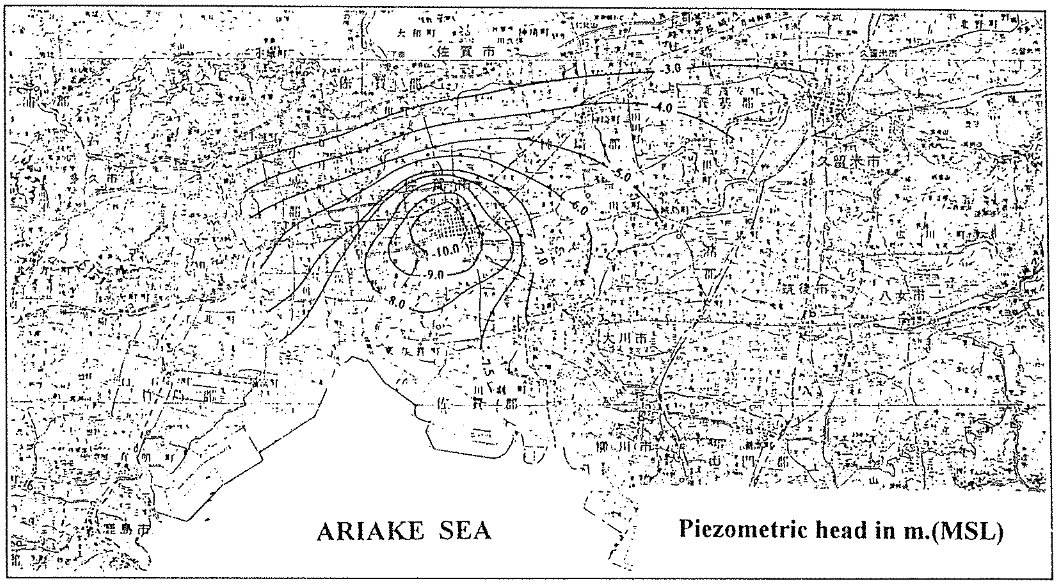


Figure 1 0.Piezometric Contour in the Lower Aquifer Computed by the Finer Model levels as required.

4) Reliable data on observed heads and groudwater extraction rates are required for model calibration.

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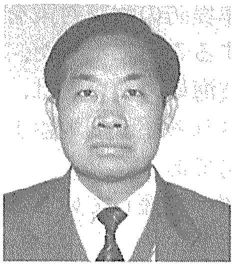

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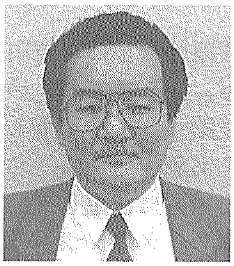

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